

AN UPDATED STATISTICAL CATCH-AT-LENGTH ASSESSMENT FOR WESTERN ATLANTIC BLUEFIN TUNA

D. S. Butterworth¹ and R. A. Rademeyer¹

SUMMARY

Butterworth and Rademeyer (2014) provided an initial Statistical Catch-at-Length (SCAL) assessment of the western population of North Atlantic bluefin tuna. The primary purpose in fitting to length- rather than to age-distribution data was to avoid the need to make use of the somewhat coarse cohort-slicing method to provide the latter. Here these analyses are updated using comparable inputs to those agreed for the 2014 updated VPA assessments.

RÉSUMÉ

Butterworth et Rademeyer (2014) fournissait une évaluation initiale de la prise par taille statistique (SCAL) des populations orientales de thon rouge de l'Atlantique Nord. L'objectif principal de l'ajustement aux données de taille, plutôt qu'aux données de distribution par âge, visait à éviter de devoir utiliser la méthode de découpage des cohortes quelque peu grossière afin de fournir cette dernière. Dans le présent document, ces analyses ont été mises à jour au moyen de données d'entrée comparables à celles des évaluations mises à jour de la VPA de 2014.

RESUMEN

Butterworth y Rademeyer (2014) proporcionaba una evaluación inicial de la captura por talla estadística (SCAL) de la población de atún rojo del Atlántico norte occidental. El propósito principal de ajustar a los datos de talla más que a los datos de distribución por edad es evitar la necesidad de utilizar el método de separación de cohortes, algo tosco, para proporcionar esta última. Estos análisis se actualizan utilizando datos de entrada comparables a los acordados para las evaluaciones mediante VPA actualizadas de 2014.

KEYWORDS

Bluefin tuna, Stock assessment, Assessment models, Catch at length

1. Introduction

Butterworth and Rademeyer, 2014 introduced a statistical catch-at-length (SCAL) approach for the assessment of the western Atlantic population of bluefin tuna. A particular purpose was to avoid the need for use of the crude cohort-slicing approach to provide catch-at-age data needed for application of the VPA assessment method conventionally applied to this resource.

The paper first describes the data used and the SCAL methodology. To the extent that comparable data and assumptions are concerned, these have been selected to attempt to duplicate similar choices for the current updated VPA assessments. This is followed by the SCAL results, and a brief discussion of their implications. It should be noted that the purpose of this paper, which is of an initial nature, is not to offer a comprehensive application of SCAL, exploring the implications of all possible associated sensitivities, but rather to provide a comparison to the VPA outputs together with a baseline for discussion towards refinement of the approach.

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2. Data and Methods

The data used for these analyses are listed in **Appendix A**.

The SCAL methodology is described in detail in **Appendix B**. This is not applied to the size structure data for all indices of abundance yet, with some still included on an age-based SCAA basis, as described in **Appendix B**. **Figure 1** shows the growth curve together with the distributions of length-at-age which are assumed; the SCAL method applied treats these as time-invariant. Note also that the assessment commences in 1970, well after exploitation of this resource commenced, so that the 1970 numbers-at-age vector is estimated in the model fitting process rather than being linked in any way to the pre-exploitation equilibrium biomass (see **Appendix B**, section B.1.4).

3. Results and Discussion

Figure 2 compares the (Base Case) SCAL results for spawning biomass and recruitment with those from the updated VPA (Continuity run). The SCAL spawning biomass estimates are quite similar to those from the VPA, though initially higher. In contrast, recruitment estimates at the start of the period considered are lower. **Table 1** indicates that the resource was above its equilibrium pre-exploitation level in 1970 – a possibility which free estimation of the starting numbers at age does not exclude, but which may also be somewhat influenced by the specification of a high steepness of $h = 0.98$ for the stock-recruitment relationship.

Figure 3 shows the stock recruitment plot (note that here steepness h was fixed at 0.98) together with the annual estimates of recruitment and spawning biomass. There is some indication of higher recruitments at the larger biomasses earlier in the period considered. The recruitment estimates for the last few years are not reliable for reasons discussed below.

The fits to the CPUE series are shown in **Figure 4**. Some are quite reasonable, but others are poor. Probably the most serious lack of fit shown for recent years is for the JLL WEST and particularly the Canadian GLS indices, which indicate appreciably higher abundances than the model can reflect.

The fits to the CAL data for the various fisheries shown in **Figure 5**. When averaged over years, the fits are broadly good, and the bubble plots for the residuals are also reasonable except for the sport fishery. A problem, however, is evident for the longline fisheries for which there is no catch of smaller fish for the last four years. This necessitates a change of selectivity for this period, but this has yet to be implemented. In these circumstances, the estimates of recruitment for the last few years will be negatively biased (as was evident in **Figure 3**). All effective selectivities-at-age for the fisheries catching the largest fish exhibit domes, though most of these do not indicate strong decreases.

Figure 6 shows similar plots to **Figure 5** for the age or length data corresponding to the catches associated with the various indices of abundance. The fits are generally good, though this may in part reflect over-parameterisation of some of the selectivity functions.

4. Concluding remarks

The analysis reported here is of an initial nature, given time constraints. Certainly improvements are required, in particular to allow for a change in the selectivity of the longline fishery over the last few years. Other matters requiring investigation are the impacts of different assumptions for the CV for the distribution of lengths-at-age, allowing estimation of the steepness parameter of the stock-recruitment relationship, more parsimonious parameterisation of the selectivity functions for the size data linked to the various indices of abundance, and a move to fit to catch-at-length throughout rather than still to catch-at-“age” for the size data linked to some of the indices of abundance. Subsequently the assessment should also be taken back further in time.

At this stage though, particularly interesting features of the assessment are the relatively good agreement with the VPA results for the spawning biomass trend, the absence of strong doming in the selectivity functions estimated, and the rather lower recruitment estimates at the start of the period considered compared to the estimates from the VPA.

Acknowledgements

We thank Matthew Loretta for kindly providing the updated VPA results for the western Atlantic Bluefin assessment.

References

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Table 1. Results for the SCAL Base Case. Biomass units are mt.

SCAL	
-lnL:overall	-3323.3
-lnL: CPUE	-15.9
-lnL: fleet CAL	-1882.3
-lnL: index CAA	-691.6
-lnL: index CAL	-743.8
-lnL: RecRes	10.1
K^{sp}	64024
B^{sp}_{1970}	79568
B^{sp}_{2013}	24585
B^{sp}_{1970}/K^{sp}	1.24
B^{sp}_{2013}/K^{sp}	0.38

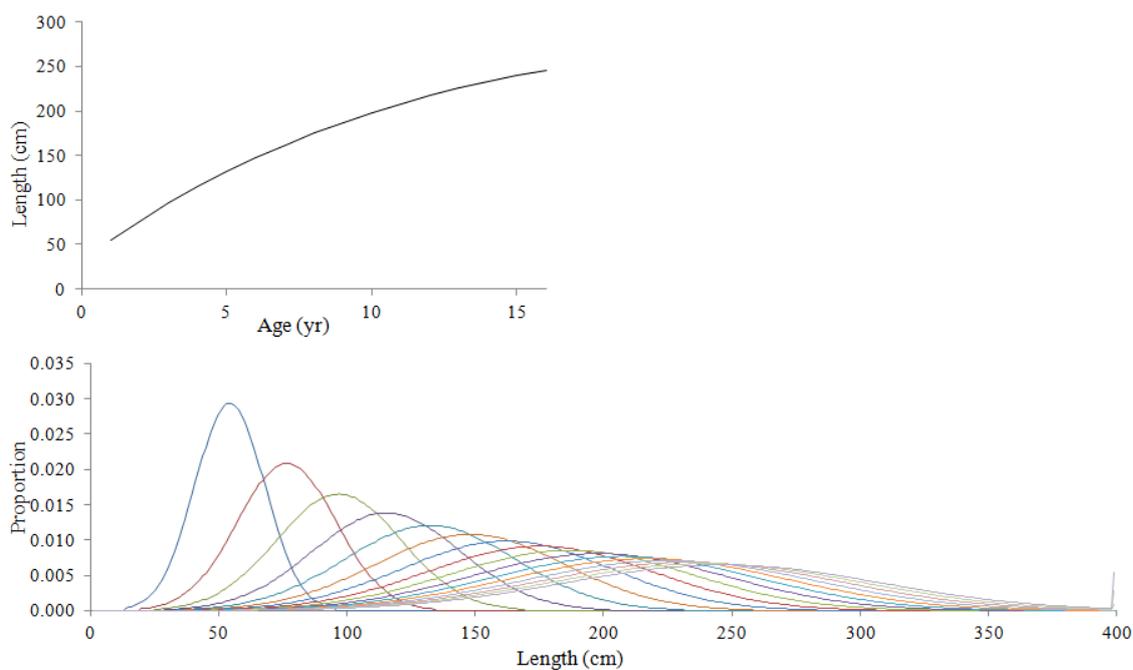


Figure 1. Von Bertalanffy growth curve and associated length-at-age distributions assumed. See **Table B1** for details of the growth curve parameters. The length-at-age distributions are assumed to be normal with CVs of 25%.

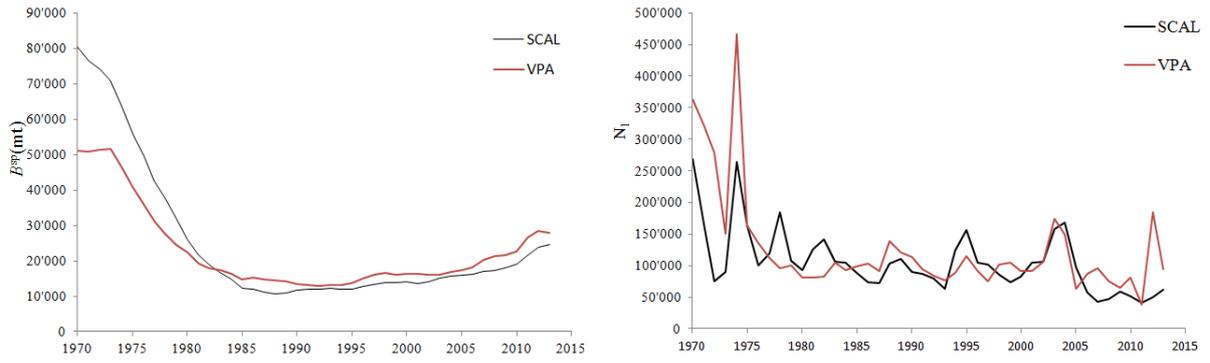


Figure 2. Spawning biomass and recruitment (number of 1-year-olds, N_1) trajectories for the SCAL Base Case and the VPA. VPA refers to 2014 Continuity Run.

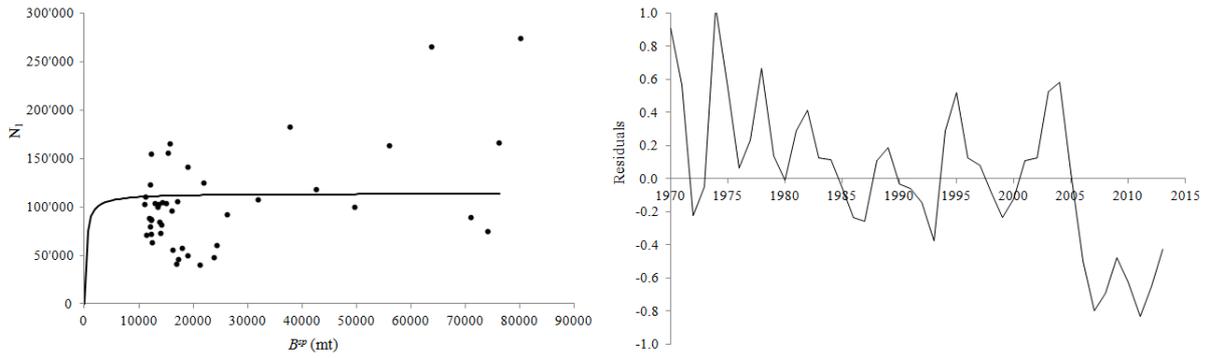


Figure 3. Stock-recruitment relationships (left-hand column) and time series of stock-recruitment residuals for the SCAL Base Case. Spawning stock biomass (SSB) is in mt.

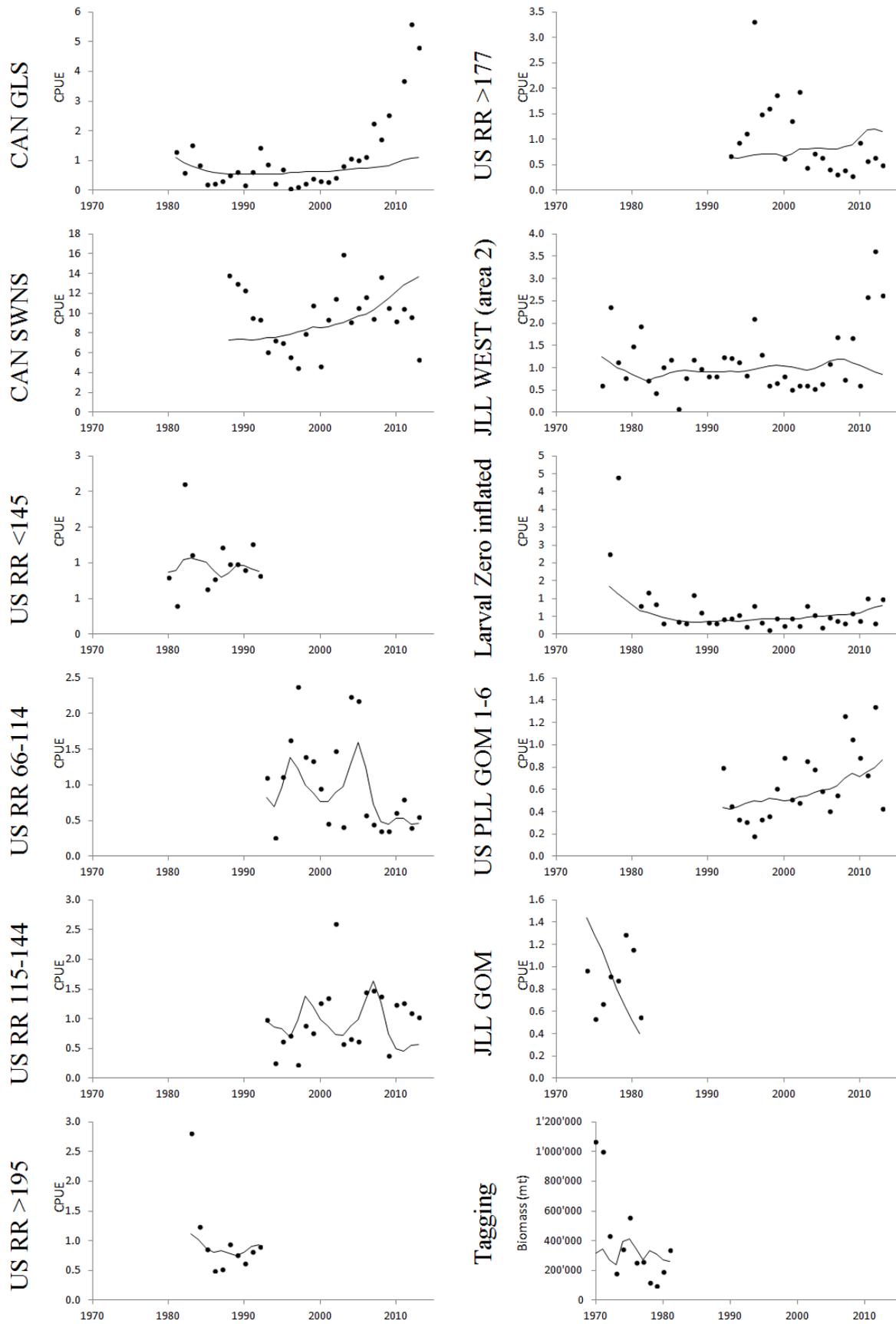


Figure 4. Fits of the SCAL Base Case to the various CPUE series.

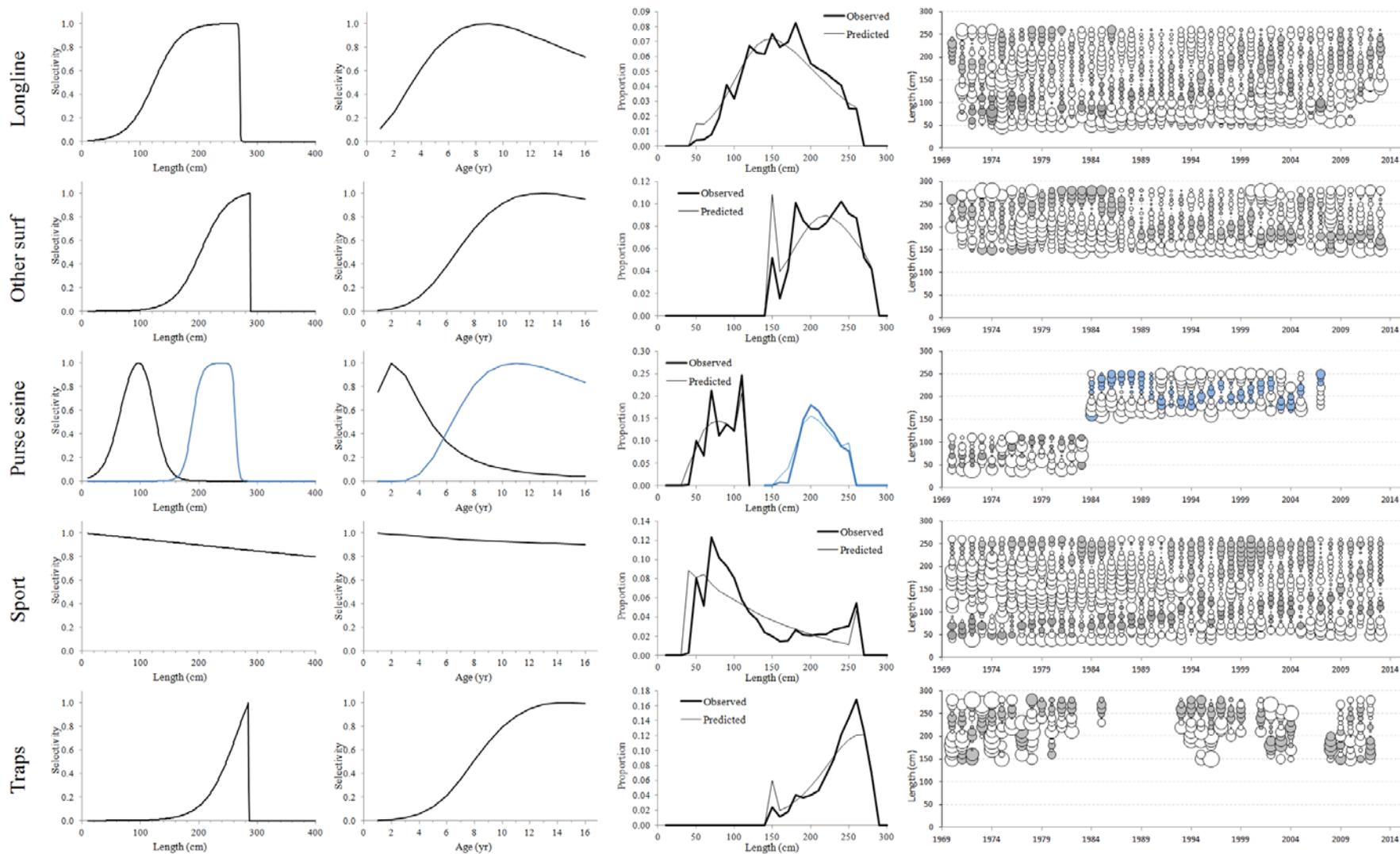


Figure 5. Estimated selectivities-at-length, the effective equivalent selectivities-at-age, fit to the CAL data (as average over all the years with data available), and bubble plots of the CAL standardised residuals for the associated fisheries for the SCAL Base Case.

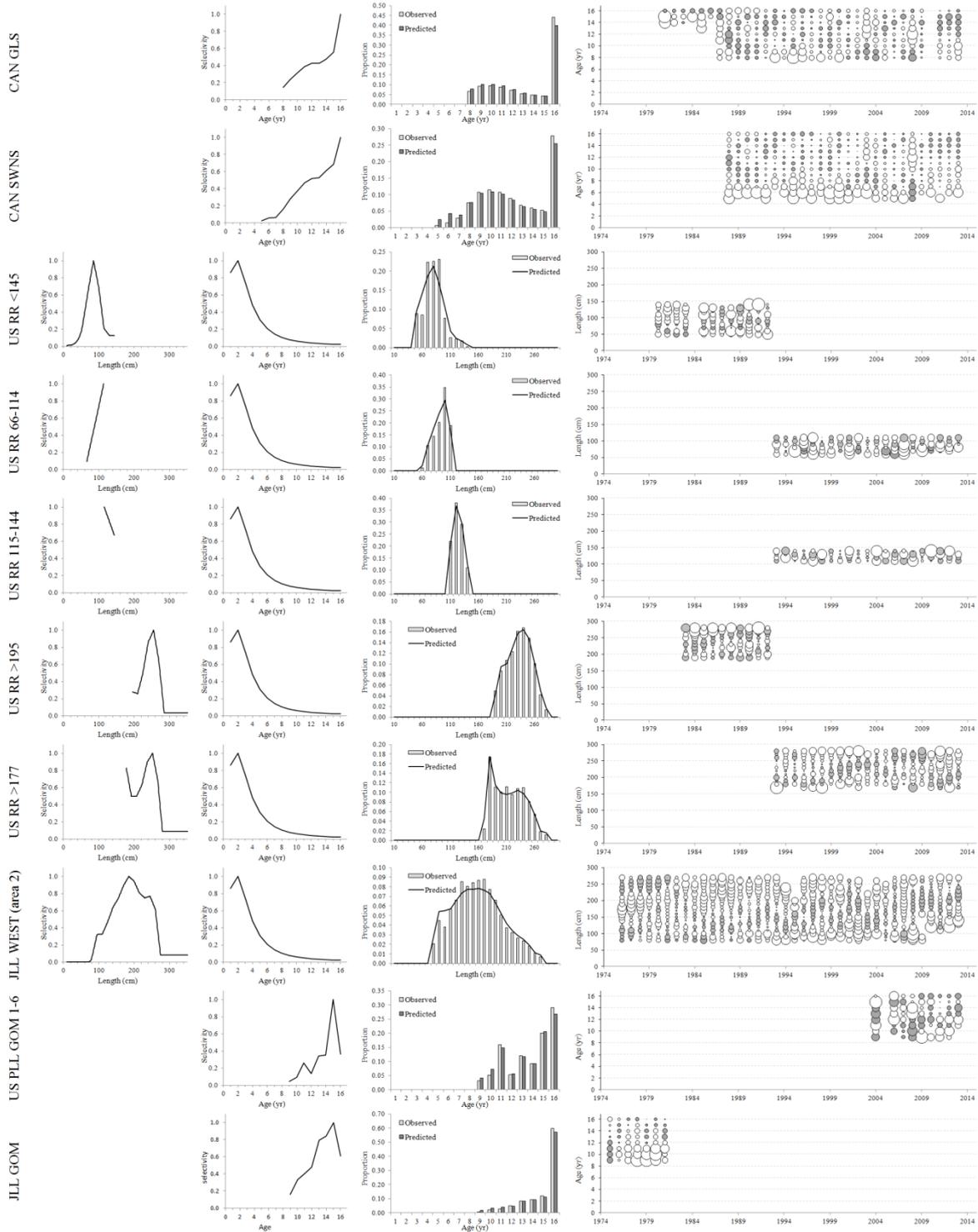


Figure 6. Estimated selectivities-at-length (where applicable), the effective equivalent selectivities-at-age, fit to the CAA/CAL data (as average over all the years with data available), and bubble plots for the CAA/CAL standardised residuals for the catches associated with indices of abundance for the SCAL Base Case. Note that for CAN GLS, CAN SWNS, US PLL GOM 1-6 and JLL GOM, the model is fit to CAA data rather than CAL data.

The data

Table A1. Catches in mt.

	Longline	Other	Purse seine	Sport	Traps
1970	268.0	83.0	4288.0	644.0	183.0
1971	1390.0	182.0	3769.0	1144.0	106.0
1972	339.0	186.0	2011.0	1354.0	58.0
1973	1127.0	115.0	1656.0	816.0	157.0
1974	946.0	256.0	960.0	2955.0	276.0
1975	1562.4	24.0	2320.0	1022.0	144.0
1976	3066.0	311.0	1582.0	752.0	172.0
1977	3753.4	194.0	1502.0	874.0	372.0
1978	3219.1	191.0	1230.0	904.0	221.0
1979	3691.0	196.0	1381.0	956.0	31.0
1980	3972.5	131.0	758.0	893.0	47.0
1981	3878.0	133.0	910.0	808.0	41.0
1982	360.0	323.0	232.0	459.0	68.0
1983	829.0	514.0	384.0	808.0	7.0
1984	823.0	377.0	401.0	676.0	3.0
1985	1229.0	293.0	377.0	750.0	20.0
1986	1272.0	166.0	360.0	518.0	0.0
1987	1237.0	156.0	367.0	726.0	17.0
1988	1473.3	425.0	383.0	601.0	14.0
1989	817.6	769.0	385.0	786.0	1.0
1990	854.1	536.0	384.0	1004.0	2.0
1991	1022.3	578.0	237.0	1083.0	0.0
1992	885.0	509.3	300.0	586.0	1.0
1993	783.0	406.0	295.0	854.0	29.0
1994	621.3	307.2	301.0	804.0	79.0
1995	602.0	384.0	249.0	1114.0	72.0
1996	713.6	436.0	245.0	1029.0	90.0
1997	537.0	293.0	250.0	1195.3	59.0
1998	887.0	342.0	249.0	1111.0	68.0
1999	1074.5	281.0	248.0	1123.4	44.5
2000	1079.5	284.4	275.2	1119.7	16.1
2001	714.7	201.9	195.9	1655.7	15.8
2002	940.0	107.5	207.7	2035.1	28.1
2003	418.1	139.3	265.4	1398.3	84.0
2004	824.8	97.1	31.8	1138.8	32.0
2005	556.2	89.1	178.3	924.5	8.4
2006	714.4	85.3	3.6	1005.1	3.0
2007	519.9	63.1	27.9	1022.9	3.6
2008	764.7	81.9	0.0	1129.9	23.0
2009	573.2	120.7	11.4	1250.6	23.5
2010	703.1	106.7	0.0	1008.9	38.8
2011	944.5	147.1	0.0	887.3	26.3
2012	701.2	117.3	1.7	916.3	16.6
2013	67.4	76.3	0.0	325.2	11.4

Table A2. Commercial fleet catch-at-length used in the SCAL.

Longline	30-	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300+	
1970	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	181	181	273	273	362	91	0	0	0	0	0	0	
1971	0	0	0	0	70	14	140	61	0	108	37	10	189	195	573	1116	1247	902	764	571	622	283	294	35	19	0	0	0	
1972	0	0	4	25	0	66	46	0	6	11	33	132	37	19	55	309	247	107	53	86	100	106	18	15	0	0	0	0	
1973	0	0	47	20	45	284	165	94	601	274	422	435	142	124	262	805	1029	509	182	360	432	105	116	97	0	0	0	0	
1974	0	20	79	735	1451	2671	485	485	1026	257	540	49	27	350	699	154	789	397	469	540	446	292	136	55	6	1	0	0	
1975	0	0	1	1	6	19	37	27	32	55	195	6	6	13	130	225	566	1154	939	1082	1312	1249	751	300	29	6	0	0	
1976	0	8	36	81	300	835	2223	2075	3494	2131	1300	419	418	195	78	137	634	731	733	1633	2843	2470	1700	882	159	8	8	4	
1977	0	0	56	126	256	414	3247	5120	5347	1728	1223	1424	1600	750	274	130	204	186	357	561	1649	2517	2439	1317	372	77	31	23	
1978	0	0	0	59	64	303	879	1133	1813	1645	1420	1735	841	788	322	220	221	195	354	580	1104	1901	2179	1662	592	200	37	6	
1979	0	0	16	40	52	202	479	420	1343	649	519	980	1660	1597	1258	731	367	301	446	848	1637	1968	1599	1058	556	192	55	12	
1980	0	0	21	70	89	274	1123	1517	1614	873	390	827	1143	1330	3812	3147	1591	663	601	727	1147	2370	2704	1505	594	192	22	8	
1981	0	7	35	344	801	1385	5091	2604	2912	2947	1608	2184	1876	1842	1690	1467	1227	554	757	815	804	1261	1081	1351	944	491	185	273	
1982	0	6	60	4	119	48	93	68	167	73	98	271	248	237	202	214	201	154	176	201	170	43	66	32	26	14	7	6	
1983	0	0	0	0	12	125	1454	615	281	252	531	529	445	454	700	661	505	415	187	230	136	96	46	27	25	9	0	17	
1984	2	0	3	12	58	543	1074	259	733	1076	724	835	991	900	415	228	371	250	217	119	214	136	95	39	50	16	2	10	
1985	10	7	17	17	701	2187	3696	663	1375	2409	1974	2662	1670	707	346	295	246	174	207	296	330	282	148	83	40	11	2	7	
1986	0	2	1	1	8	43	134	671	1097	1117	1606	680	496	891	637	422	317	359	195	300	321	446	487	353	168	101	85	2	
1987	45	4	2	33	25	143	534	645	1420	1556	2268	2186	2064	1219	916	936	570	401	232	146	154	166	121	90	51	30	24	12	
1988	48	0	12	75	92	460	965	1125	2802	2754	1798	1757	2327	1933	1376	956	484	353	199	238	213	160	203	94	31	23	9	25	
1989	64	1	1	31	10	41	270	159	349	844	716	536	692	747	892	820	396	318	205	218	207	140	122	66	32	21	0	0	
1990	68	0	0	35	48	328	492	82	299	967	1025	721	761	646	465	616	413	419	343	235	143	159	145	71	43	15	15	7	
1991	77	0	0	37	32	38	262	161	438	975	955	1192	1003	1020	1052	666	373	373	357	225	234	217	131	64	42	55	10	0	
1992	92	3	7	44	11	67	261	81	320	966	445	421	1053	748	860	902	503	329	205	274	217	160	133	64	27	31	0	6	
1993	34	0	0	16	4	23	170	156	935	714	733	1161	1051	789	586	649	573	291	204	143	102	89	93	56	18	3	0	0	
1994	53	0	0	25	3	11	232	350	939	1555	984	956	691	455	430	544	348	170	132	90	53	87	57	26	18	6	2	0	
1995	110	0	0	52	59	3	260	30	75	757	789	299	1716	1639	436	234	287	135	77	76	76	49	37	32	21	7	0	12	
1996	102	0	0	52	9	157	202	179	473	1081	855	400	607	499	841	641	278	176	250	250	174	138	84	55	47	6	0	0	
1997	70	0	0	33	3	5	99	80	73	583	365	424	647	561	682	664	478	186	100	95	105	75	41	18	19	3	0	0	
1998	51	0	0	24	2	16	135	191	305	662	278	445	828	562	433	870	1115	965	328	185	184	54	118	26	19	8	0	0	
1999	60	0	0	29	3	4	115	23	104	633	450	822	655	437	520	906	764	460	578	670	411	248	145	88	63	3	11	0	
2000	46	0	0	22	2	3	378	151	309	1781	1459	582	1846	1614	934	847	411	201	174	206	142	120	68	65	32	6	5	0	
2001	12	0	0	8	54	6	28	28	9	104	47	136	218	482	962	691	363	397	316	376	214	147	91	60	15	2	1	14	
2002	0	0	0	8	23	6	13	6	27	56	60	106	201	356	870	1028	1159	977	541	332	174	270	52	28	10	0	0	0	
2003	0	0	0	9	3	4	11	12	10	242	170	167	389	182	136	331	191	203	222	227	150	205	66	19	27	0	0	0	
2004	0	0	0	4	3	4	0	22	169	858	927	1427	1011	929	740	453	329	220	213	340	162	237	56	48	14	10	0	0	
2005	0	0	0	6	22	124	361	422	317	310	498	285	417	536	460	422	278	234	385	284	163	194	61	23	26	2	0	0	
2006	0	0	0	0	87	359	76	104	527	334	264	952	727	446	701	973	704	561	455	266	252	416	129	45	27	18	1	7	
2007	0	0	0	0	3	49	1809	2115	486	452	636	508	591	303	213	259	155	168	241	203	132	133	54	35	20	1	1	10	
2008	0	0	0	1	0	4	47	105	180	225	579	441	337	531	799	905	678	521	481	332	198	229	82	16	34	13	15	8	
2009	0	0	0	2	0	0	12	0	0	39	0	23	250	57	147	438	378	315	360	185	160	215	97	41	33	6	4	20	
2010	0	0	0	8	63	0	67	17	64	55	137	28	337	97	265	738	484	457	545	336	224	231	106	66	39	10	6	8	
2011	0	0	0	0	0	0	0	0	0	41	551	590	270	342	502	931	1136	790	306	355	371	255	227	132	51	33	7	1	5
2012	0	0	0	0	0	0	0	0	0	1	123	9	259	389	140	444	376	643	570	342	176	269	133	66	54	12	1	22	
2013	0	0	0	0	0	0	0	0	0	0	15	4	127	56	72	249	120	272	640	513	295	260	123	45	36	11	1	5	

Other	30-	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300+	
1970	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	25	52	32	20	27	130	0	0	0	0	
1971	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	8	8	0	8	8	105	168	235	102	36	8	0	0	0
1972	0	0	0	3	0	9	6	0	1	3	6	17	11	7	8	39	32	24	36	116	222	163	81	31	8	0	0	0	
1973	0	0	2	1	2	12	7	4	25	11	17	18	6	5	13	33	46	26	13	28	44	67	92	63	17	1	0	0	
1974	0	1	4	36	71	131	24	24	51	13	27	3	1	18	37	8	41	26	29	82	178	258	201	60	4	2	0	0	
1975	0	0	0	0	0	1	1	1	1	1	3	0	0	0	2	3	8	16	13	15	18	17	10	4	1	0	0	0	
1976	0	0	1	2	18	17	51	39	42	28	25	11	8	8	4	11	28	47	52	99	144	184	174	101	35	8	4	2	
1977	0	0	5	21	8	10	15	28	23	10	6	5	6	3	2	1	1	2	4	6	27	72	129	161	86	25	1	0	
1978	0	0	0	6	2	6	8	11	17	15	13	16	8	7	5	2	3	5	7	41	111	175	145	52	8	3	1	0	
1979	0	0	0	0	1	2	3	3	10	5	4	7	12	11	9	5	5	10	7	31	67	112	108	130	102	34			

Table A2. Continued.

Purse seinr	30-	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300+	
1970	0	2502	55630	6208	77805	27229	89488	25635	4549	13654	5293	1992	975	384	73	92	24	0	0	0	0	0	0	0	0	0	0	0	
1971	0	531	47526	19660	118397	9569	10049	28661	30564	14676	248	106	460	424	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1972	0	15	13521	30319	46349	40998	15773	14292	419	2053	1569	2610	205	0	132	146	0	0	0	0	0	0	0	0	0	0	0	0	
1973	0	0	3992	1926	59657	9435	6492	21530	446	5021	1185	524	1695	118	0	26	13	0	0	0	0	0	0	0	0	0	0	0	
1974	0	310	17907	2767	8354	8280	11688	3921	580	3378	1585	1546	679	125	75	32	13	7	11	15	39	49	26	11	3	0	0	0	
1975	0	0	25899	5406	134796	8571	1475	3937	7821	6096	603	220	518	205	64	12	62	96	81	75	123	212	207	97	12	0	0	0	
1976	54	0	3783	171	13333	4634	56984	7720	29	0	0	0	0	0	0	6	10	23	29	67	88	123	136	61	21	4	0	0	
1977	0	12	401	355	9980	8008	420	2744	3556	17346	2781	1372	436	692	179	1	9	11	10	47	126	165	112	57	15	4	0	0	
1978	0	0	381	3533	1717	4807	6047	10566	2214	1930	3767	3152	2437	339	17	55	60	35	13	7	12	18	33	25	0	1	1	0	
1979	0	0	0	44	3186	3123	5054	8489	1930	10032	4324	97	217	193	56	2	0	18	12	6	44	253	450	233	91	12	6	0	
1980	227	0	464	1584	1187	6897	6030	3769	3307	2863	2246	237	101	21	5	21	286	103	77	53	18	11	25	27	15	7	3	0	
1981	0	17	601	2127	3022	3999	801	5761	1109	552	845	392	114	59	50	151	761	1212	584	236	115	60	4	4	11	3	0	0	
1982	0	24	213	590	57	352	131	533	246	24	9	0	2	17	29	66	78	200	301	249	79	38	1	4	0	1	0	0	
1983	0	0	1285	612	17	0	63	20	0	9	0	0	0	16	13	36	215	187	288	412	352	148	57	18	10	0	0	1	
1984	0	0	10	62	120	87	0	0	0	0	0	0	9	2	8	29	106	228	401	329	368	153	61	30	5	0	0	0	
1985	0	0	0	0	0	0	0	0	0	1	0	0	0	0	6	17	39	106	215	330	321	360	146	28	9	1	0	0	
1986	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	23	38	78	198	292	405	245	68	13	3	0	0	0	
1987	0	0	0	0	0	0	0	0	0	0	0	0	0	1	7	36	141	233	201	191	235	266	207	64	17	2	1	1	
1988	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	13	111	220	266	211	211	252	208	82	27	3	2	0	
1989	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	27	103	199	286	271	244	227	160	48	11	4	1	0	
1990	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	12	169	399	328	272	219	193	125	47	15	1	0	0	
1991	0	0	1	2	3	0	0	0	0	0	0	1	1	5	25	281	462	295	122	36	37	15	12	1	4	0	0	0	
1992	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	142	139	239	293	202	157	85	62	24	2	0	0	0	
1993	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11	279	736	437	118	79	36	13	5	0	0	0	0	0	
1994	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	422	397	384	295	111	52	20	13	3	0	0	0	0	
1995	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	279	588	241	137	67	35	9	7	2	0	0	0	0	
1996	0	0	0	0	0	0	0	0	0	0	0	0	0	1	5	48	94	216	384	218	116	62	27	12	2	0	0	0	
1997	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22	235	174	181	247	230	123	42	17	1	0	0	0	0	
1998	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	27	333	447	225	161	182	69	16	7	1	0	0	0	
1999	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	36	240	515	396	164	67	36	12	1	0	0	0	0	
2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	30	288	269	342	300	163	63	14	6	0	0	0	0	0	
2001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19	53	129	253	271	199	66	19	0	1	0	0	0	0	
2002	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	107	125	46	122	251	255	142	43	11	3	0	0	0	
2003	0	0	0	0	0	0	0	0	0	0	0	0	0	0	40	527	525	334	157	105	104	48	25	13	2	0	0	0	
2004	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	69	65	70	29	6	0	0	0	0	0	0	0	0	
2005	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	11	44	122	272	259	115	52	35	25	8	0	0	0	
2006	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	13	2	4	1	2	1	0	0	0	0	0	0	0	
2007	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	7	7	9	5	19	18	22	14	4	0	0	0	0	
2009	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	54	25	6	0	0	0	0	0	0	0	0	0	0	0
2012	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	8	0	0	0	0	0	0	0	0	0	0	0	0
2013	0	0	0	0	0	0	0	0	0	0	0	0	0	20	33	109	58	57	30	5	0	1	1	0	0	0	0	0	0

Sport	30-	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300+
1970	0	270	4852	644	3549	329	449	0	15	15	0	0	23	23	3	141	121	269	557	409	394	131	53	54	0	0	3	
1971	0	0	6803	6803	4082	777	0	0	0	0	8	120	45	68	83	15	53	403	899	1001	659	297	69	8	8	8	0	0
1972	0	2	1427	3319	3813	4189	1713	1216	16	230	176	294	23	15	16	89	44	15	161	474	1034	1120	867	528	197	23	1	0
1973	0	0	192	80	2490	356	238	874	18	233	47	37	125	18	10	35	95	60	62	157	358	603	686	467	117	20	0	0
1974	0	0	34107	784	588	980	1176	0	0	0	0	0	3	0	0	1	6	401	18	38	1688	1284	3510	1492	766	31	4	1
1975	9	153	12321	1364	1002	1508	9	68	149	145	30	0	9	26	12	9	33	18	43	83	277	652	812	590	331	88	8	9
1976	0	0	1020	298	463	397	210	7	10	46	30	12	6	0	7	12	28	40	95	138	255	427	498	414	119	16	0	0
1977	17	16	306	221	3368	465	115	325	64	49	21	0	4	4	5	0	6	1	6	23	76	193	488	672	557	156	13	1
1978	46	4	301	1185	2157	1335	119	131	25	15	11	6	2	8	0	0	21	30	12	12	30	209	444	742	583	250	40	3
1979	7	0	564	2113	925	3015	242	345	30	23	13	3	7	30	103	75	18	25	39	69	166	284	397	568	520	279	42	5
1980	5	0	228	695	905	4058	369	206	116	129	56	40	5	27	13	16	88	50	38	70	86	123	295	564	548	316	78	4
1981	0	13	238	2005	963	1134	297	231	92	13	13	20	26	0	0	8	73	177	228	157	92	92	191	340	494	372	132	13
1982	0	42	2282	443	1304	1613	393	326	101	59	42	34	68	40	23	28	82	108	253	236	67	82	128	153	137	47	37	25
1983	0	180	773	643	1865	534	419	424	195	35	21	25	20	67	60	50	159	89	136	193	330	366	402	404	277	69	36	30
1984	23	11	381	550	4040	1392	485	714	149	201	134	67	66	103	64	70	105	142	215	285	422	445	259	150	69	11	16	4
1985	0	58	382	131	3585	1749																						

Table A2. Continued.

Traps	30-	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300+		
1970	0	0	0	0	0	0	0	0	0	0	0	1	1	2	1	1	2	10	36	79	133	182	100	64	29	9	0	1		
1971	0	0	0	0	0	0	0	4	5	13	1	8	6	13	3	2	1	3	19	37	88	89	80	35	13	0	0	0		
1972	0	0	0	0	0	1	0	0	0	4	5	6	26	21	5	0	11	35	35	25	27	22	25	28	10	1	0	0		
1973	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22	52	181	155	164	27	8	3	0	0	0		
1974	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	3	11	30	166	267	278	149	40	3	0	0		
1975	0	0	0	0	0	0	0	0	0	0	1	0	0	3	0	3	0	0	3	10	41	80	123	95	56	17	5	2		
1976	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	6	50	131	172	90	12	0	0	0		
1977	0	0	0	0	0	0	0	0	0	0	0	0	0	3	30	133	313	655	682	277	18	0	0	0	0	0	0	0		
1978	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	2	2	0	1	0	2	12	18	67	122	128	110	36		
1979	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	15	17	30	8	1	0		
1980	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	2	6	3	7	1	4	6	6	22	30	21	13	4	0	
1981	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	6	7	29	33	13	1	0		
1982	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	5	4	25	49	50	19	2	0	0		
1983	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	5	6	3	0	0	0		
1984	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	2	1	0	0	0		
1985	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	5	9	14	16	5	0	0	0		
1987	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	3	3	0	2	5	6	8	5	9	5	0		
1988	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	3	3	0	2	4	5	7	4	7	4	0	0		
1989	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	0	0	0	0		
1990	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	1	0	0	0	0		
1992	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	
1993	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	5	10	20	27	12	4	0	0	0		
1994	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4	6	21	41	62	49	25	0	0	0		
1995	0	0	0	0	0	0	0	0	0	0	0	0	3	2	0	0	1	0	1	8	9	19	30	56	35	22	5	4		
1996	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	3	2	1	12	29	35	36	64	44	33	15	1	0		
1997	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	1	2	15	30	46	46	17	0	0	0		
1998	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	10	17	23	46	43	36	22	7	6	3	0		
1999	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	8	14	29	34	35	7	7	2	0	0		
2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	6	11	9	9	9	3	3	0	0		
2001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	5	8	15	10	8	2	2	0	0	0		
2002	0	0	0	0	0	0	0	0	0	0	0	0	2	16	51	27	10	14	16	17	8	2	0	1	0	0	0	0		
2003	0	0	0	0	0	0	0	0	0	0	0	13	0	13	110	131	78	10	28	53	27	13	13	0	0	0	0	0	0	
2004	0	0	0	0	0	0	0	0	0	5	0	1	2	3	1	8	6	14	23	51	30	10	1	0	0	0	0	0	0	
2005	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	0	1	5	10	7	1	4	1	2	0	0	0	0	0	
2006	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	2	3	0	0	0	2	1	2	1	0	0	0	0	0	
2007	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	6	0	1	0	0	0	0	
2008	0	0	0	0	0	0	0	0	0	0	0	0	8	11	42	54	44	27	0	0	0	0	0	0	0	0	0	0	0	0
2009	0	0	0	0	0	0	0	0	0	0	9	17	2	3	10	7	2	16	6	8	8	9	13	5	0	0	0	0	0	
2010	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4	1	9	7	9	18	27	19	21	14	5	0	0	0	0	
2011	0	0	0	0	0	0	0	0	0	0	3	0	5	0	2	0	1	3	5	3	11	15	16	22	6	3	0	0	0	
2012	0	0	0	0	0	0	0	0	0	0	1	0	5	5	6	7	4	5	9	7	7	6	5	6	2	0	0	0	0	
2013	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	5	6	4	1	8	7	7	2	0	1	0	0	0	

Table A3. CPUE (relative abundance) series used.

	CAN GLS	CAN SWNS	US RR<145	US RR 66-114	US RR 115-144	US RR>195	US RR>177	JLL WEST (area 2)	Larval zero inflated	US PLL GOM 1-6	JLL GOM	Tagging
Units	Numbers	Numbers	Numbers	Numbers	Numbers	Numbers	Numbers	Numbers	Biomass	Numbers	Numbers	Numbers
1970	-	-	-	-	-	-	-	-	-	-	-	1065132
1971	-	-	-	-	-	-	-	-	-	-	-	1001624
1972	-	-	-	-	-	-	-	-	-	-	-	431955
1973	-	-	-	-	-	-	-	-	-	-	-	183616
1974	-	-	-	-	-	-	-	-	-	-	0.968	341589
1975	-	-	-	-	-	-	-	-	-	-	0.534	554596
1976	-	-	-	-	-	-	-	2.250	-	-	0.666	253265
1977	-	-	-	-	-	-	-	4.390	2.250	-	0.913	257385
1978	-	-	-	-	-	-	-	-	4.390	-	0.876	121110
1979	-	-	-	-	-	-	-	-	-	-	1.287	98815
1980	-	-	0.799	-	-	-	-	0.810	-	-	1.158	192541
1981	1.320	-	0.399	-	-	-	-	1.180	0.810	-	0.553	337995
1982	0.600	-	2.102	-	-	-	-	0.840	1.180	-	-	-
1983	1.540	-	1.114	-	-	2.805	-	0.310	0.840	-	-	-
1984	0.850	-	-	-	-	1.246	-	-	0.310	-	-	-
1985	0.210	-	0.630	-	-	0.857	-	0.350	-	-	-	-
1986	0.240	-	0.778	-	-	0.503	-	0.310	0.350	-	-	-
1987	0.320	-	1.219	-	-	0.529	-	1.110	0.310	-	-	-
1988	0.530	13.860	0.988	-	-	0.941	-	0.620	1.110	-	-	-
1989	0.650	13.030	0.988	-	-	0.763	-	0.330	0.620	-	-	-
1990	0.190	12.320	0.904	-	-	0.626	-	0.300	0.330	-	-	-
1991	0.650	9.510	1.261	-	-	0.820	-	0.420	0.300	-	-	-
1992	1.450	9.410	0.820	-	-	0.910	-	0.440	0.420	0.80	-	-
1993	0.900	6.090	-	1.100	0.990	-	0.690	0.540	0.440	0.45	-	-
1994	0.250	7.280	-	0.260	0.260	-	0.940	0.220	0.540	0.33	-	-
1995	0.720	7.040	-	1.110	0.630	-	1.130	0.790	0.220	0.31	-	-
1996	0.080	5.560	-	1.630	0.730	-	3.330	0.330	0.790	0.18	-	-
1997	0.130	4.480	-	2.370	0.240	-	1.500	0.110	0.330	0.33	-	-
1998	0.240	7.950	-	1.390	0.900	-	1.620	0.460	0.110	0.36	-	-
1999	0.420	10.820	-	1.330	0.770	-	1.880	0.250	0.460	0.61	-	-
2000	0.320	4.660	-	0.950	1.270	-	0.630	0.460	0.250	0.89	-	-
2001	0.290	9.370	-	0.460	1.360	-	1.380	0.240	0.460	0.51	-	-
2002	0.450	11.490	-	1.480	2.600	-	1.940	0.790	0.240	0.48	-	-
2003	0.830	15.900	-	0.410	0.590	-	0.450	0.550	0.790	0.86	-	-
2004	1.080	9.150	-	2.230	0.670	-	0.740	0.180	0.550	0.78	-	-
2005	1.040	10.550	-	2.180	0.630	-	0.650	0.470	0.180	0.59	-	-
2006	1.140	11.660	-	0.580	1.460	-	0.430	0.390	0.470	0.41	-	-
2007	2.280	9.480	-	0.450	1.480	-	0.330	0.310	0.390	0.55	-	-
2008	1.740	13.650	-	0.350	1.380	-	0.400	0.580	0.310	1.26	-	-
2009	2.560	10.570	-	0.350	0.390	-	0.290	0.390	0.580	1.05	-	-
2010	-	9.180	-	0.610	1.240	-	0.940	1.020	0.390	0.89	-	-
2011	3.700	10.430	-	0.800	1.270	-	0.590	0.300	1.020	0.73	-	-
2012	5.620	9.660	-	0.400	1.110	-	0.650	0.980	0.300	1.34	-	-
2013	4.810	5.340	-	0.550	1.040	-	0.500	2.620	0.980	0.43	-	-

Table A4: Catches-at-age associated with the CPUE series used in the SCAA.

In the interests of keeping this document shorter, these data have not been listed below, but can be provided by the authors if required.

Table A5: Catches-at-length associated with the CPUE series used in the SCAL.

In the interests of keeping this document shorter, these data have not been listed below, but can be provided by the authors if required.

The Statistical Catch-at-Age Model

The text following sets out the equations and other general specifications of the SCAA followed by details of the contributions to the (penalised) log-likelihood function from the different sources of data available and assumptions concerning the stock-recruitment relationship. Quasi-Newton minimization is then applied to minimize the total negative log-likelihood function to estimate parameter values (the package AD Model Builder™ (Fournier *et al.*, 2011) is used for this purpose). The description below includes more options than used in this paper, but they have been included here for completeness as they may be used in later extensions.

B.1. Population dynamics

B.1.1 Numbers-at-age

The resource dynamics are modelled by the following set of population dynamics equations:

$$N_{y+1,1} = R_{y+1} \quad (\text{B1})$$

$$N_{y+1,a+1} = \left(N_{y,a} e^{-M_a/2} - \sum_f C_{y,a}^f \right) e^{-M_a/2} \quad \text{for } 1 \leq a \leq m-2 \quad (\text{B2})$$

$$N_{y+1,m} = \left(N_{y,m-1} e^{-M_{m-1}/2} - \sum_f C_{y,m-1}^f \right) e^{-M_{m-1}/2} + \left(N_{y,m} e^{-M_m/2} - \sum_f C_{y,m}^f \right) e^{-M_m/2} \quad (\text{B3})$$

where

$N_{y,a}$ is the number of fish of age a at the start of year y (which refers to a calendar year),

R_y is the recruitment (number of 1-year-old fish) at the start of year y ,

M_a denotes the natural mortality rate for fish of age a ,

$C_{y,a}^f$ is the predicted number of fish of age a caught in year y by fleet f , and

m is the maximum age considered (taken to be a plus-group).

B.1.2 Recruitment

The number of recruits (i.e. new 1-year olds) at the start of year y is assumed to be related to the spawning stock size (i.e. the biomass of mature fish) at the mid-point of the preceding year by a Beverton-Holt stock-recruitment relationship, allowing for annual fluctuation about the deterministic relationship:

$$R_y = \frac{\alpha B_{y-1}^{\text{sp}}}{\beta + B_{y-1}^{\text{sp}}} e^{(\zeta_y - (\sigma_R)^2/2)} \quad (\text{B4})$$

where

α and β are spawning biomass-recruitment relationship parameters,

ζ_y reflects fluctuation about the expected recruitment for year y , which is assumed to be normally distributed with standard deviation σ_R (which is input in the applications considered here); these residuals are treated as estimable parameters in the model fitting process.

B_y^{sp} is the spawning biomass in year y , computed as:

$$B_y^{\text{sp}} = \sum_{a=0}^m f_{y,a} w_{y,a}^{\text{sp}} N_{y,a} e^{-M_a \frac{T^s}{12}} \quad (\text{B5})$$

where spawning for the stocks under consideration is taken to occur T^s months after the start of the year (here $T^s = 6$) and some natural mortality has therefore occurred,

$w_{y,a}^{sp}$ is the mass of fish of age a during spawning, and
 $f_{y,a}$ is the proportion of fish of age a that are mature.

The estimation is carried out in terms of parameters h (steepness) and pre-exploitation equilibrium spawning biomass K^{sp} , which are related to the parameters α and β of equation B4 by:

$$R_0 = K^{sp} / SPR_0 \quad (B6)$$

where

$$SPR_0 = \sum_{a=1}^{m-1} f_a w_a^{sp} e^{-\frac{T_s}{12} \sum_{a=1}^{a-1} M_a} + f_m w_m^{sp} \frac{e^{-\frac{T_s}{12} \sum_{a=1}^{m-1} M_a}}{1 - e^{-\frac{T_s}{12} M_m}} \quad (B7)$$

and

$$\alpha = 4hR_0 / (5h - 1)$$

$$\beta = K^{sp} (1 - h) / (5h - 1) \quad (B8)$$

In the implementation considered here h is fixed on input.

B.1.3 Total catch and catches-at-age

The total catch by mass in year y is given by:

$$C_y = \sum_f \sum_{a=0}^m w_{y,a}^f C_{y,a}^f = \sum_f \sum_{a=0}^m w_{y,a}^f N_{y,a} e^{-M_a/2} S_{y,a}^f F_y^f \quad (B9)$$

where

$w_{y,a}^f$ denotes the mass of fish of age a landed in year y by fleet f ,

$C_{y,a}^f$ is the catch-at-age, i.e. the number of fish of age a , caught in year y by fleet f ,

$S_{y,a}^f$ is the commercial selectivity of fleet f (i.e. combination of availability and vulnerability to fishing gear) at age a for year y ; when $S_{y,a} = 1$, the age-class a is said to be fully selected, and

F_y^f is the proportion of a fully selected age class that is fished by fleet f .

The model estimate of the mid-year exploitable (“available”) component of biomass for fleet f is calculated by converting the numbers-at-age into mid-year mass-at-age (using the individual weights of the landed fish) and applying natural and fishing mortality for half the year:

$$B_y^f = \sum_{a=0}^m w_{y,a}^f S_{y,a}^f N_{y,a} e^{-M_a/2} (1 - S_{y,a}^f F_y^f / 2) \quad (B10)$$

B.1.4 Initial conditions

For the first year (y_0) considered in the model (here 1970), the numbers-at-age are estimated directly for ages 1 to a^{est} , with a parameter ϕ which mimicking recent average fishing mortality for ages above a^{est} , ($a^{est}=9$ here), i.e.

$$N_{y_0,a} = N_{start,a} \quad \text{for } 1 \leq a \leq a^{est} \quad (B11)$$

and

$$N_{start,a} = N_{start,a-1} e^{-M_{a-1}} (1 - \phi S_{a-1}) \quad \text{for } a^{est} < a \leq m - 1 \quad (B12)$$

$$N_{\text{start},m} = N_{\text{start},m-1} e^{-M_{m-1}} (1 - \phi S_{m-1}) / (1 - e^{-M_m} (1 - \phi S_m)) \quad (\text{B13})$$

B.2. The (penalised) likelihood function

The model can be fit to (a subset of) CPUE, and commercial catch-at-age or catch-at-length data to estimate model parameters (which may include residuals about the stock-recruitment function, facilitated through the incorporation of a penalty function described below). Contributions by each of these to the negative of the (penalised) log-likelihood ($-\ln L$) are as follows.

B.2.1 CPUE relative abundance data

The likelihood is calculated assuming that an observed CPUE index for a particular fishing fleet is log-normally distributed about its expected value:

$$I_y^i = \hat{I}_y^i \exp(\varepsilon_y^i) \quad \text{or} \quad \varepsilon_y^i = \ln(I_y^i) - \ln(\hat{I}_y^i) \quad (\text{B14})$$

where

I_y^i is the CPUE biomass or abundance index for year y for gear/flag combination i ,

$\hat{I}_y^i = \hat{q}^i \sum_{a=1}^m w_{y,a}^i S_{y,a}^i N_{y,a} e^{-M_a/2} (1 - S_{y,a}^i F_y^i / 2)$ is the corresponding model estimate of biomass or

$\hat{I}_y^f = \hat{q}^f \sum_{a=1}^m S_{y,a}^f N_{y,a} e^{-M_a/2} (1 - S_{y,a}^f F_y^f / 2)$ is the corresponding model estimate of abundance,

\hat{q}^i is the constant of proportionality (catchability) for the CPUE series, and

ε_y^i from $N(0, (\sigma^{CPUE})^2)$.

The contribution of the CPUE data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\ln L^{CPUE} = \sum_y \left\{ \ln \left(\sqrt{(\sigma^{CPUE})^2 + (\sigma_{Add}^i)^2} \right) + \frac{(\varepsilon_y^i)^2}{2[(\sigma^{CPUE})^2 + (\sigma_{Add}^i)^2]} \right\} \quad (\text{B15})$$

where

σ^{CPUE} is the standard deviation of the residuals for the logarithm of the indices,

σ_{Add}^i is the square root of the additional variance for the CPUE series, which can be estimated in the model fitting procedure but has been set to zero in the applications considered here.

σ^{CPUE} is estimated in the fitting procedure by its maximum likelihood value:

$$\sigma^{CPUE} = \sqrt{\sum_i \sum_y (\ln(I_y^i) - \ln(\hat{I}_y^i))^2 / \sum_i \sum_y 1}$$

The catchability coefficient q^i for CPUE index i is estimated by its maximum likelihood value:

$$\ln \hat{q}^i = 1/n_i \sum_y (\ln I_y^i - \ln \hat{B}_y^{\text{ex}}) \quad (\text{B16})$$

B.2.3 Commercial catches-at-age

The contribution of the catch-at-age data to the negative of the log-likelihood function under the assumption of an “adjusted” lognormal error distribution is given by:

$$-\ln L^{CAA} = w_{CAA} \sum_f \sum_y \sum_a \left[\ln \left(\sigma_{\text{com}}^f / \sqrt{p_{y,a}^f} \right) + p_{y,a}^f \left(\ln p_{y,a}^f - \ln \hat{p}_{y,a}^f \right)^2 / 2 \left(\sigma_{\text{com}}^f \right)^2 \right] \quad (\text{B17})$$

where

$p_{y,a}^f = C_{y,a}^f / \sum_{a'} C_{y,a'}^f$ is the observed proportion of fish caught in year y by fleet f that are of age a ,

$\hat{p}_{y,a}^f = \hat{C}_{y,a}^f / \sum_{a'} \hat{C}_{y,a'}^f$ is the model-predicted proportion of fish caught in year y by fleet f that are of age a ,

where

$$\hat{C}_{y,a}^f = N_{y,a} S_{y,a}^f F_y^f e^{-M_a/2} \quad (\text{B18})$$

and

σ_{com}^f is the standard deviation associated with the catch-at-age data, which is estimated in the fitting procedure by:

$$\hat{\sigma}_{\text{com}}^f = \sqrt{\sum_y \sum_a p_{y,a}^f \left(\ln p_{y,a}^f - \ln \hat{p}_{y,a}^f \right)^2 / \sum_y \sum_a 1} \quad (\text{B19})$$

The log-normal error distribution underlying equation (B17) (Punt and Kennedy, 1997) is chosen on the grounds that (assuming no ageing error) variability is likely dominated by a combination of interannual variation in the distribution of fishing effort, and fluctuations (partly as a consequence of such variations) in selectivity-at-age, which suggests that the assumption of a constant coefficient of variation is appropriate. However, for ages poorly represented in the sample, sampling variability considerations must at some stage start to dominate the variance. To take this into account in a simple manner, motivated by binomial distribution properties, the observed proportions are used for weighting so that undue importance is not attached to data based upon a few samples only.

Commercial catches-at-age are incorporated in the likelihood function using equation (B17), for which the summation over age a is taken from age a_{minus} (considered as a minus group) to a_{plus} (a plus group).

The w_{CAA} weighting factor may be set to a value less than 1 to downweight the contribution of the catch-at-age data (which tend to be positively correlated between adjacent ages) to the overall negative log-likelihood compared to that of the CPUE data. Here, $w_{CAA} = 0.1$

In instance where catch-at-age data corresponding to a particular CPUE index are available, the data are treated in exactly the same manner as described above, with a specific selectivity S_a^i estimated for that index.

B.2.4 Commercial catches-at-length

Commercial catches-at-length are incorporated in the likelihood function in the same manner as the catches-at-age. When the model is fit to catches-at-length, selectivity is estimated as a function of length and then converted to selectivity-at-age:

$$S_{y,a}^f = \sum_l S_{y,l}^f A_{a,l} \quad (\text{B20})$$

where $A_{a,l}$ is the proportion of fish of age a that fall in the length group l (i.e., $\sum_l A_{a,l} = 1$ for all ages).

The matrix $A_{a,l}$ is calculated under the assumption that length-at-age is normally distributed about a mean given by the von Bertalanffy equation, i.e.:

$$L_a \sim N\left[L_\infty\left(1 - e^{-\kappa(a-t_0)}\right), \theta_a^2\right] \quad (\text{B21})$$

where

θ_a is the standard deviation of length-at-age a , which is modelled to be proportional to the expected length-at-age a , i.e.:

$$\theta_a = \beta L_\infty\left(1 - e^{-\kappa(a-t_0)}\right) \quad (\text{B22})$$

with β fixed here to 0.25.

Furthermore, in the model fitting to CAL, the weights-at-age used to compute the CPUE indices are weighted by the selectivity for the corresponding fleet:

$$\tilde{w}_{y,a}^i = \sum_l S_{y,l}^f w_l A_{a,l} / S_{a,l}^i \quad (\text{B23})$$

$\tilde{w}_{y,a}^i$ is the selectivity-weighted mid-year weight-at-age a for fleet f and year y ; and

w_l is the weight of fish of length l ;

The following term (replacing equation B15) is then added to the negative log-likelihood:

$$-\ln L^{\text{CAL}} = w_{\text{len}} \sum_f \sum_y \sum_l \left[\ln\left(\sigma_{\text{len}}^f / \sqrt{p_{y,l}^f}\right) + p_{y,l}^f \left(\ln p_{y,l}^f - \ln \hat{p}_{y,l}^f \right)^2 / 2\left(\sigma_{\text{len}}^f\right)^2 \right] \quad (\text{B24})$$

The w_{len} weighting factor may be set to a value less than 1 to downweight the contribution of the catch-at-length data (which tend to be positively correlated between adjacent length groups) to the overall negative log-likelihood compared to that of the CPUE data. Here, $w_{\text{len}} = 0.05$

B.2.5 Stock-recruitment function residuals

The stock-recruitment residuals are assumed to be log-normally distributed. Thus, the contribution of the recruitment residuals to the negative of the (now penalised) log-likelihood function is given by:

$$-\ln L^{\text{pen}} = \sum_{y=y_1+1}^{y_2} \left[\zeta_y^2 / 2\sigma_R^2 \right] \quad (\text{B25})$$

where

ζ_y is the recruitment residual for year y , which is estimated for year y_1 to y_2 (see equation (B4)), and

σ_R is the standard deviation of the log-residuals, which is input (here $\sigma_R=0.6$).

B.3. Model parameters

The model input parameters are given in **Table B1**.

Table B1. Input parameters (Length-weight, von Bertalanffy growth, maturity and natural mortality at age to age 16 from Anon., 2013). Length, weight and time units are cm, gm and yr respectively.

Model plus group (m)	16
Length-weight	$a=0.00002861, b=2.929$
von Bertalanffy growth	$\beta=0.089, L_{\text{inf}}=315, t_0=-1.13$
Maturity-at-age	100% maturity at age 9
Natural mortality	0.14 yr^{-1}
Stock-recruitment	Beverton-Holt, $h=0.98, \sigma_R=0.6$

B.4.2 Fishing selectivity

Commercial fishing selectivities-at-length are estimated using a four parameters double-logistic form:

$$S_l = \left(1 + e^{-a_1(l-b_1)}\right)^{-1} \left[1 - \left(1 + e^{-a_2(l-b_2)}\right)^{-1}\right] \quad (\text{B26})$$

Selectivities-at-length for the indices are estimated separately for specified lengths from l_{minus} to l_{plus} , assuming linear changes from the lowest to the highest length for each length group. For the indices for which catch-at-age is used rather than catch-at-length, the selectivities-at-age are estimated directly for ages a_{minus} to a_{plus} . The selectivity is assumed to stay flat after l_{plus} if not otherwise specified. The selectivity can differ between fixed periods. Details of the fishing selectivities used are shown in **Table B2**.

Table B2. Details of the selectivities estimated. The * indicates an actual minimum or maximum rather than a minus- or plus-group.

	SCAA/SCAL				Number of parameters estimated	Comments
	a_{minus} (yr)	a_{plus} (yr)	l_{minus} (cm)	l_{plus} (cm)		
Commercial fleet:						
Longline			50	260	4	
Other			150	280	4	
Purse seine			40	110	4	First selectivity period: 1950-1983
			160	250	4	Second selectivity period: 1984-present
Sport			40	260	4	
Traps			150	280	4	
CPUE indices:						
CAN GLS	8	16			3	
CAN SWNS	5	16			6	
US RR<145			55	144*	5	
US RR 66-114			66*	114*	3	
US RR 115-144			115*	144*	2	
US RR>195			196*	280	6	
US RR>177			178*	280	7	
JLL WEST (area 2)			80	270	13	
Larval zero inflated	9*	16			-	Assume spawning biomass, i.e. age 9+
US PLL GOM 1-6	9*	16			7	
JLL GOM	9*	16			7	
Tagging	1*	3*			-	Flat selectivity for ages 1 to 3